

**Exercise 10C**

1 Simplify each of the following expressions:

a $1 - \cos^2 \frac{1}{2}\theta$

b $5 \sin^2 3\theta + 5 \cos^2 3\theta$

c $\sin^2 A - 1$

d $\frac{\sin \theta}{\tan \theta}$

e $\frac{\sqrt{1 - \cos^2 x}}{\cos x}$

f $\frac{\sqrt{1 - \cos^2 3A}}{\sqrt{1 - \sin^2 3A}}$

g $(1 + \sin x)^2 + (1 - \sin x)^2 + 2 \cos^2 x$

h $\sin^4 \theta + \sin^2 \theta \cos^2 \theta$

i $\sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta$

2 Given that $2 \sin \theta = 3 \cos \theta$, find the value of $\tan \theta$.

3 Given that $\sin x \cos y = 3 \cos x \sin y$, express $\tan x$ in terms of $\tan y$.

4 Express in terms of $\sin \theta$ only:

a $\cos^2 \theta$

b $\tan^2 \theta$

c $\cos \theta \tan \theta$

d $\frac{\cos \theta}{\tan \theta}$

e $(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)$

P 5 Using the identities $\sin^2 A + \cos^2 A \equiv 1$ and/or $\tan A = \frac{\sin A}{\cos A}$ ($\cos A \neq 0$), prove that:

a $(\sin \theta + \cos \theta)^2 \equiv 1 + 2 \sin \theta \cos \theta$

b $\frac{1}{\cos \theta} - \cos \theta \equiv \sin \theta \tan \theta$

c $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$

d $\cos^2 A - \sin^2 A \equiv 1 - 2 \sin^2 A$

e $(2 \sin \theta - \cos \theta)^2 + (\sin \theta + 2 \cos \theta)^2 \equiv 5$

f $2 - (\sin \theta - \cos \theta)^2 \equiv (\sin \theta + \cos \theta)^2$

g $\sin^2 x \cos^2 y - \cos^2 x \sin^2 y \equiv \sin^2 x - \sin^2 y$

6 Find, without using your calculator, the values of:

a $\sin \theta$ and $\cos \theta$, given that $\tan \theta = \frac{5}{12}$ and θ is acute.

b $\sin \theta$ and $\tan \theta$, given that $\cos \theta = -\frac{3}{5}$ and θ is obtuse.

c $\cos \theta$ and $\tan \theta$, given that $\sin \theta = -\frac{7}{35}$ and $270^\circ < \theta < 360^\circ$.

7 Given that $\sin \theta = \frac{2}{3}$ and that θ is obtuse, find the exact value of: **a** $\cos \theta$ **b** $\tan \theta$

8 Given that $\tan \theta = -\sqrt{3}$ and that θ is reflex, find the exact value of: **a** $\sin \theta$ **b** $\cos \theta$

9 Given that $\cos \theta = \frac{3}{4}$ and that θ is reflex, find the exact value of: **a** $\sin \theta$ **b** $\tan \theta$

(P) 10 In each of the following, eliminate θ to give an equation relating x and y :

a $x = \sin \theta, y = \cos \theta$ **b** $x = \sin \theta, y = 2 \cos \theta$

c $x = \sin \theta, y = \cos^2 \theta$ **d** $x = \sin \theta, y = \tan \theta$

e $x = \sin \theta + \cos \theta, y = \cos \theta - \sin \theta$

Problem-solving

In part **e** find expressions for $x + y$ and $x - y$.

(E/P) 11 The diagram shows the triangle ABC with $AB = 12$ cm, $BC = 8$ cm and $AC = 10$ cm.

a Show that $\cos B = \frac{9}{16}$

(3 marks)

b Hence find the exact value of $\sin B$.

(2 marks)



Hint

Use the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ ← Section 9.1

(E/P) 12 The diagram shows triangle PQR with $PR = 8$ cm, $QR = 6$ cm and angle $QPR = 30^\circ$.

a Show that $\sin Q = \frac{2}{3}$

(3 marks)

b Given that Q is obtuse, find the exact value of $\cos Q$

(2 marks)

